

	Answer the following questions:	Time: 3 Hours
1	<p>(a) Test the series: (i) $\sum_{n=1}^{\infty} \frac{n}{n+2^n}$ (ii) $\sum_{n=1}^{\infty} (-1)^n \frac{\sin n}{n^2}$</p> <p>(b) Find the interval of convergence of the series: $\sum_{n=1}^{\infty} n \left(\frac{x-1}{2}\right)^n$</p> <p>(c) Show that the envelope of the curves: $x \cos \alpha + y \sin \alpha = 4$ is the circle: $x^2 + y^2 = 16$</p>	
2	<p>(a) Find the extrema of the function: $f(x,y) = x^2 + 2y^2 - 2x + 8y + 4$</p> <p>(b) Show that the orthogonal trajectories of the circles: $x^2 + (y-a)^2 = a^2$ are the circles $(x-b)^2 + y^2 = b^2$</p> <p>(c) Solve the equation: $(e^x + y \sin x)dx + (2y - \cos x)y = 0$</p>	
3	<p>Solve the differential equations:</p> <p>(a) $y' + 2xy = 4x$ (b) $y'' + y = 3 \sin 2x + x^2$</p> <p>(c) $(x^2 D^2 - 2xD - 4)y = x^4 + 4$</p>	
4	<p>(a) Prove that: If $f(x,y,z)$ is a function, then $\text{curl grad } f = \vec{0}$.</p> <p>(b) Verify Stoke's theorem for the vector: $\vec{U} = 2y\vec{i} + 3x\vec{j} - z^2\vec{k}$ through the semi-sphere $x^2 + y^2 + z^2 = 9, z \geq 0$</p>	
5	<p>(a) Find the sum of the series: $\sin \theta + \sin 2\theta + \sin 3\theta + \dots$</p> <p>(b) Find the bilinear transformation that maps the three points: $-1, i, 1$ in z-plane onto the three points $0, i, \infty$ in w-plane.</p> <p>(c) Evaluate the integrals: (i) $\int_C \frac{5z^2}{(z-3)(z-8)} dz$ (ii) $\int_C \frac{e^z}{(z^2-7z+6)^2} dz$</p> <p>where C is the ellipse $z-4 + z+4 = 10$</p>	

Good Luck

Dr. M.H. Eid